Selling in Advance to Loss Averse Consumers

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Abstract

This paper examines the influence of information on market performance in an advance purchase setting. Information reduces the risk that an advance purchase results in a mismatch between consumer preferences and product characteristics. However, information may also raise the number of advance purchases by increasing a firm’s incentive to offer advance purchase discounts. Our main result shows that accounting for consumer loss aversion leads to a qualitative change in the assessment of policies aiming to improve consumers’ information at the advance purchase stage. Under monopoly information can be detrimental both for efficiency and consumer surplus whereas under competition information is beneficial because it mitigates inter-temporal business stealing.

JEL classification: D43, D83, L13, L41

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1 Introduction

In a variety of markets, consumers purchase products in advance, that is, with imperfect knowledge about their preferences. Advance selling is standard in ticketing markets (e.g.
transportation, entertainment, accommodation) and, with the emergence of e-commerce, has become an important marketing tool in the form of pre-orders for new to-be-released products (e.g. music albums, video games, “en primeur” wine). Recent trends towards a customer-based financing of entrepreneurial activity (crowd-funding) have widened the scope for advance selling by allowing products to be sold even prior to their development.

Advance selling benefits firms in various ways as it allows them to obtain more accurate demand forecasts (Moe and Fader, 2002), manage their capacity (Liu and van Ryzin, 2008), screen their customers inter-temporally (Nocke et al., 2011), or steal future business from their rivals (Möller and Watanabe, 2016). However, from a regulatory viewpoint, there exist important concerns about the effects of advance selling on consumer surplus and market efficiency. Because it induces consumers to make less-informed purchases, advance selling entails the risk of a mismatch between consumer preferences and product characteristics, and the resulting welfare losses can be substantial.¹

Regulatory policies that improve consumers’ information at the advance purchase stage seem to offer a way to mitigate this problem. For example, in ticketing markets, consumers’ information could be influenced via adjustments to the duration of the advance-purchase period. In a pre-order setting, firms could be obliged to provide samples, beta-versions, or more detailed descriptions of their products’ characteristics. The direct effect of such policies is to reduce the likelihood that an advance-purchase results in a mismatch. Less clear, however, is what indirect effect such policies would have on a firm’s incentive to sell in advance. In particular, improving consumers’ information could have the adverse effect of increasing the number of advance purchases via its influence on a firm’s pricing policy.² The objective of this paper is to improve our understanding of the role of consumers’ information in markets with an advance purchase option.

In light of the trade-off between the potential money-savings from an advance purchase

¹Lazarev (2013) calculates that in the US airline industry the welfare loss associated with inter-temporal price discrimination amounts to 6% of overall welfare.

²In 2013, the British rail company Cross Country applied to the Secretary of State for Transport for permission to reduce the minimum time period required for a ticket to qualify as an “Advance fare”. The Secretary of State approved the application but noted that “it remains to be seen how this might change wider ticket buying habits or how it might impact overall price levels [...].” (Rail fares and ticketing review, UK Department for Transport, available at https://www.gov.uk/government/consultations/rail-fares-and-ticketing-review.)
discount and the risk of a sub-optimal product match, the consumers’ attitude towards risk can be expected to constitute an important element for the description of market demand. Unfortunately, the modeling of risk preferences has proven as rather elusive in the advance purchase literature. Moreover, it has been argued that for risky choices involving relatively small stakes, such as the choice between alternative (differentiated) products, risk aversion does not offer an adequate description of individual decision making (Rabin, 2000). A more suitable description consists of the concept of *loss aversion* introduced by Kahneman and Tversky (1979) which assumes consumers to evaluate outcomes in relation to a reference point and to weigh losses more heavily than gains. In this paper, we provide a tractable description of consumer risk attitudes in advance purchase markets by allowing consumers to be loss averse with respect to a reference point, given by their expectations (Kőzegi and Rabin, 2006, 2007).

We argue that consumer loss aversion has a major impact on the way in which advance selling depends on information and hence on our assessment of policies aiming to improve consumers’ information. Importantly, loss aversion alters our assessment in opposite directions depending on whether advance selling is used to screen customers (monopoly) or to steal a rival’s business (competition).

In our model, consumers face the choice between two differentiated products which can be purchased during an advance purchase period and a consumption period. In the advance purchase period each consumer obtains an informative but imperfect signal about the identity of his preferred product. All uncertainty becomes resolved before consumption takes place. The quality of information (i.e. the precision of the idiosyncratic signals) is the same for all consumers, providing us with an exogenous measure of the market’s information. Consumers differ in their “choosiness” measuring the additional utility a consumer derives from consuming his preferred rather than his non-preferred product. Products are offered by a single monopolistic supplier (monopoly) or by two rival firms (competition). Firms commit to a pricing policy consisting of a regular price and an advance purchase discount.

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3So called expectation-based loss aversion is well documented, with evidence both from the field (Card and Dahl, 2011; Crawford and Meng, 2011; Pope and Schweitzer, 2011; Allen et al., 2017) and the lab (Abeler et al., 2011; Ericson and Fuster, 2011; Gill and Prowse, 2012; Karle et al., 2015) and has been applied to a large range of economic settings. See Grubb (2015) for a recent overview with focus on applications in Industrial Organization.
As a motivating example, consider a traveler choosing between a morning and an evening flight. Suppose that flying at the “correct” (“incorrect”) time gives a match value of 200 (160) and that both departure times are equally likely to be correct. On the day of departure, the morning and the evening flight are priced at $70 and $50 respectively, whereas when purchased in advance, a discount of $10 applies to both. At the advance purchase stage, the traveler expects a match value of 180 from both flights and opting for the cheaper evening flight gives an expected utility of $180 – ($50 – $10) = 140. Waiting to guarantee the purchase of the correct flight, gives a match value of 200 at an expected price of $60. Hence, in the absence of loss aversion, the traveler is just indifferent between purchasing in advance and waiting.

Expectation-based loss aversion à la Kőszegi and Rabin (2006, 2007) postulates that, in addition to the intrinsic utilities derived above, the traveler derives extra utility/disutility from gains/losses with respect to his expectations. Gains and losses are evaluated separately in the match-value and the money-dimension and losses weigh more, say twice as much as gains. Expecting a match value of 180 from an advance purchase, the traveler experiences a gain/loss of size 20 depending on whether his purchase turns out to be correct, i.e. loss aversion reduces the traveler’s expected utility by $\frac{1}{2}(200 – 180) + \frac{1}{2}(160 – 180) \cdot 2 = -10$. Similarly, when postponing his purchase, the traveler expects to pay $60 for the correct flight and experiences either a money gain or a money loss of size $10, leading to a reduction of his expected utility by $\frac{1}{2}($60 – $50) + \frac{1}{2}($60 – $70) \cdot 2 = -$5$. As the disutility from gains/losses in the match-value dimension turns out to be larger than the disutility from gains/losses in the money-dimension, in this example, loss aversion tilts the traveler’s preference towards a late purchase.

While the above example highlights the effects of loss aversion on a consumer’s willingness to purchase in advance for given prices, consumer loss aversion also affects a firm’s pricing policy. In the first part of the paper we determine the combined effect of loss aversion on the inter-temporal allocation of sales. We demonstrate, both for a monopoly and the case of competition, that accounting for loss aversion changes the way in which advance selling depends on the quality of consumers’ information.

For a monopolist, advance selling serves the purpose of screening customers. An advance
purchase discount segments the market by inducing less-choosy consumers to purchase in advance. In the absence of loss aversion, the fraction of consumers a monopolist induces to purchase in advance turns out to be independent of the quality of consumers’ information. Accounting for loss aversion provides us with the new insight that in a monopolistic market, advance selling is more frequent in an informed market than in an uninformed market. Information policies aiming at a reduction of allocative inefficiencies in monopolistic advance purchase markets must therefore be assessed as less effective or even detrimental when loss aversion is taken into account.

Under competition, a very different picture emerges. Competing firms offer advance purchase discounts in order to secure the demand of consumers who, after learning their preferences, might prefer a rival’s product. We show that, in the absence of loss aversion, advance selling is more frequent in an informed market than in an uninformed market. Loss aversion increases advance selling in uninformed markets but reduces advance selling in informed markets. As a consequence, information policies aiming at a reduction of allocative inefficiencies in competitive advance purchase markets must be assessed as more effective when loss aversion is taken into account. Accounting for loss aversion offers the insight that improving consumer information can be beneficial because it mitigates inter-temporal business stealing.

In the second part of the paper we turn our attention to the influence of loss aversion on prices and consumer surplus. We find that consumer loss aversion has an anti-competitive effect on prices but a pro-competitive effect on discounts. Under competition, prices are increasing with information because better informed consumers differentiate more strongly between products. Accounting for loss aversion reveals that prices can be increasing with information also under monopoly. Higher prices lead to a reduction in consumer surplus. Hence, we can conclude that under monopoly information can be detrimental not only for efficiency but also for consumer surplus.

While our results about the inter-temporal allocation of sales (efficiency) hold very generally, i.e. for a large class of distributions of consumer types, our results about pricing (consumer surplus) assume types to be distributed uniformly. This assumption is standard in the literature on price-competition with loss-averse consumers which we discuss below. In
light of this literature, it is surprising that in our model the direction of price-effects are independent of whether loss aversion acts only in the money or the taste dimension or receives equal weight in both.

**Related Literature.** This paper contributes to our understanding of advance purchase markets. Originating from the marketing and operations research literature (Xie and Shugan, 2001; McCardle et al., 2004; Tang et al., 2004) with its focus on capacity planning and revenue management, the economic literature has emphasized inter-temporal price discrimination and business stealing as potential explanations for the prevalence of advance selling in markets characterized by demand uncertainty. Although the consumers’ attitude towards risk constitutes an important determinant of the timing of purchase, the existing literature on advance purchase markets has mostly abstracted from this issue by focusing on risk-neutral consumers with standard preferences.

There are two notable exceptions. Zhao and Stecke’s (2010) model of a monopolistic pre-order market assumes that a certain group of consumers incur a disutility of constant, exogenous size when their surplus from placing a pre-order turns out to be negative. They show that the firm can benefit from segmenting the market by inducing this group of “loss-averse” consumers to postpone their purchase. We differ from Zhao and Stecke (2010) in allowing reference points to be idiosyncratic (given by expectations) and (dis-) utilities to depend on the size of the deviations from the reference point. Moreover, our focus on the consumers’ risk of buying the wrong product enables a comparison of the case of a monopoly with the case of competition. An important insight of our analysis is that consumer loss aversion can have very different effects when pre-orders are used to steal business rather than to segment a market.

Nasiry and Popescu (2012) consider advance selling when consumers perceive regret. In their model, a monopolist sells a single product to ex-ante identical consumers with uncertain valuations who may experience two sorts of regret. Early buyers incur a disutility when their valuation turns out to be below the price they paid in advance (action-regret). In contrast, late

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buyers incur a disutility when they should have bought the seller’s product at a lower price in advance (*inaction-regret*). Hence, a consumer’s reference point consists of his optimal rather than his expected outcome. In Nasiry and Popescu (2012), the consumers’ inter-temporal preferences and their influence on firm conduct depend on the relative weight of action-regret versus inaction-regret. They coincide with the preferences of a consumer with standard preferences when both types of regret are equally important. In our setting, the effects of loss aversion in money and loss aversion in taste on the consumers’ propensity to purchase in advance are similarly opposed. However, in our model the effect of consumers’ non-standard preference on firm behavior is unambiguous. This enables us to derive predictions for market performance that depend exclusively on observable market characteristics, like the number of firms in the industry, or the quality of the consumers’ information.

Loss aversion and its effect on competition has been considered for markets *without* an advance purchase option.\(^5\) Heidhues and Kőszeği (2008) show that in markets for inspection goods, loss aversion can explain the frequently observed tendency for (not necessarily identical) firms to charge a common “focal” price, leading to a reduction in price-variation. In contrast, the effect of loss aversion on price *levels* has been found to be indeterminate. For a similar setting, Karle and Peitz (2014) show that whether loss aversion leads to an increase or a decrease in prices depends on the relative strength of loss aversion in the taste- and in the money-dimension. Loss aversion in taste turns out to have an anti-competitive effect whereas the effect of loss aversion in money is pro-competitive. While, in these articles, consumers form expectations based on a consumption plan (confirmed in equilibrium), at the (unique and exogenous) time of purchase each product’s price and match value are well known. In an advance purchase setting, the consumers’ timing of purchase and its influence on the types of gains or losses experienced adds an important new element. Although seemingly more complicated, our setting allows for the strong conclusion that prices are increasing in loss aversion, independently of whether loss aversion acts in the money- or the taste-dimension, and that loss aversion increases (inter-temporal) price variation.

Both, Heidhues and Kőszeği (2008) and Karle and Peitz (2014), base their analysis of

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loss aversion on the Hotelling model of differentiated price competition. Möller and Watanabe (2016) have extended the Hotelling model by allowing for an advance purchase period, making their framework the natural starting point for our investigation of the effects of loss aversion on advance purchase markets. For a uniform distribution of consumer types, Möller and Watanabe (2016) find that advance selling is more frequent in informed markets than in uninformed markets. We show that this insight is valid more generally when consumers have standard preferences but becomes reversed when consumers are loss averse.

From a regulatory perspective, the main implications of our findings are with respect to the assessment of policies aiming to improve consumers’ information in advance purchase markets. The need for such policies has been emphasized by a recent literature on firms’ disclosure incentives. While information about vertical product characteristics (quality) is likely to be revealed due to unraveling (Milgrom, 1981), it has been argued that firms may lack incentives to disclose information regarding horizontal product features (Sun, 2011; Koessler and Renault, 2012; Celik, 2014; Janssen and Tetrytnikova, 2016). More specifically, in a pre-order setting, Chu and Zhang (2011) find that a monopolistic firm may supply some information or none, but will never reveal all information at the advance purchase stage. If firms lack disclosure incentives then there is room for policies improving consumers’ information and the immediate question is under what conditions such policies are desirable.

2 Setup

There are two products, A and B, and a continuum of consumers with mass two. Products are differentiated (horizontally). Consumers have unit demands. A consumer obtains the value $s + \frac{1}{2} \sigma$ from consuming his preferred product and $s - \frac{1}{2} \sigma$ from consuming his non-preferred product. The parameter $s > 0$ measures a consumer’s average consumption value. It is assumed to be the same for all consumers. We make the standard assumption that $s$ is sufficiently large for the market to be covered. Consumers differ in their choosiness $\sigma \in [0, 1]$. In the eyes of more choosy consumers, differences in the products’ characteristics weigh more heavily. Choosiness $\sigma$ is distributed with continuous and strictly positive density $f$ and cumulative distribution function $F$. We assume that $f$ has an increasing hazard rate,
i.e. $\frac{f}{1-F}$ is non-decreasing.\textsuperscript{6} To keep the model symmetric, we assume that for each level of choosiness, the mass of consumers whose preferred product is $A$ is identical to the mass of consumers whose preferred product is $B$.

There are two periods; an advance purchase period (1) and a consumption period (2).\textsuperscript{7} In the advance purchase period each consumer receives a private signal about the identity of his preferred product. The product indicated by the signal will be denoted as the consumer’s favorite product. In the consumption period, all consumers learn the identity of their preferred product. The quality of information is the same for all consumers. It is given by the signal’s precision, $\gamma \in (\frac{1}{2}, 1)$, measuring the probability with which a consumer’s favorite turns out to be his preferred product.

We distinguish between two market structures. Under competition there are two firms $j \in \{A, B\}$ each offering one product. Under monopoly, both products are offered by a single firm. For simplicity we abstract from production costs. Firms can commit to a price level $p_j$ and an advance purchase discount $z_j$.\textsuperscript{8} While a consumer is required to pay $p_j$ for product $j$ during the consumption period, in the advance purchase period he pays only $p_j - z_j$.

A consumer’s overall utility consists of his intrinsic utility, given by the difference between his product’s consumption value and its price, and a gain-loss utility. Consumers are loss averse in that losses loom larger than gains. In particular, we make the standard assumption that gains and losses are weighted by $\eta$ and $\lambda \eta$, respectively, with $\eta > 0$ denoting the diversion from standard preferences and $\lambda > 1$ measuring the consumers’ degree of loss aversion. To abbreviate notation we define $l \equiv \eta(\lambda - 1) > 0$. We make the standard assumption of no-dominance of gain-loss utility by requiring that $l < 2$ (see for example Herweg et al. (2010)).\textsuperscript{9} Following Köszegi and Rabin (2006) we assume that gains and losses are evaluated

\textsuperscript{6}This requirement is relatively mild. Every log-concave distribution has an increasing hazard rate and log-concavity is satisfied by most commonly used densities. For examples see Bagnoli and Bergstrom (2005). Trivially, the hazard rate of $f$ is increasing when more choosy consumers are (weakly) more frequent than less choosy ones, i.e. when $f$ is non-decreasing.

\textsuperscript{7}Although simplistic, a two-period approach resonates well with the temporal pattern of sales in pre-order markets, where most transactions take place right before and right after the release date, as found by Hui et al. (2008) for the case of DVDs.

\textsuperscript{8}In pre-order settings, firms commonly announce their product’s release price in advance. In ticketing markets, the repeated nature of transactions provides firms with an incentive not to lower their prices close to the consumption date. For a discussion of the non-commitment case, see Möller and Watanabe (2016).

\textsuperscript{9}This assumption guarantees uniqueness of equilibrium in the limiting case of a completely uninformed
separately in the taste and the money dimension, with the consumer’s expectations serving as the reference point. After observing their signal and the firms’ pricing policies, consumers decide whether to purchase in period 1 or in period 2. In order to do so, consumers form expectations about the payments they will make (when second-period prices differ across firms) and the consumption value they will receive. If a consumer buys in period 1, his payment is deterministic but match value is random. Hence, the consumer’s gain-loss utility originates from potential differences between expected and realized consumption values. In contrast, a consumer who waits until period 2 in order to guarantee the purchase of his preferred product faces uncertainty with respect to his payment when firms price their products differently. Hence, in this case, the consumer’s gain-loss utility stems from the difference between the price the consumers expects to pay (for the product that he turns out to prefer) and the price he actually pays. In our setting, the timing of purchase provides for a natural separation between the taste- and the money-dimension of loss aversion.

3 Demand

In this section we consider the consumers’ purchase decision, for given pricing policies \((p_A, z_A)\) and \((p_B, z_B)\). Consumers need to determine whether to buy in advance or whether to postpone their purchase until the consumption period. They also need to choose between products \(A\) and \(B\). Besides the potential differences in their choosiness, consumers differ with respect to the realization of their first period signal. Half of the consumers favor product \(A\) whereas the other half favor product \(B\). We will denote these two types of consumers as \(A\)-types and \(B\)-types respectively. In order to simplify the exposition, we focus our determination of demand on the case where product \(A\)’s pricing is more aggressive than product \(B\)’s in that \(p_A \leq p_B\) and \(z_A \geq z_B\).

In order to gain a better understanding of the separate effects of loss aversion in money and loss aversion in taste, in this section we denote the loss aversion parameter \(l\) in the money and taste dimensions as \(l_m > 0\) and \(l_t > 0\), respectively.

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market but has no qualitative effect on our results. For details see proof of Proposition 2.
3.1 Product choice

Given that \( p_A - z_A \leq p_B - z_B \), for \( A \)-type consumers product \( A \) is both less expensive and more likely to match the consumer’s preferences. Hence \( A \)-type consumers prefer buying \( A \) over buying \( B \) in period 1 independently of their level of choosiness \( \sigma \). In contrast, \( B \)-types face a trade-off between buying the cheaper product \( A \) and buying their favorite product \( B \).

If a \( B \)-type consumer with choosiness \( \sigma \) buys his favorite product \( B \) in period 1 he expects a match value of size \( s + \gamma\sigma - (1 - \gamma)\sigma = s + (\gamma - \frac{1}{2})\sigma \). With probability \( \gamma \) he will be lucky obtaining the actual match value \( s + \sigma \) leading to a taste-gain with respect to the reference point of size \( \eta(s + \sigma - [s + (\gamma - \frac{1}{2})\sigma]) = \eta(1 - \gamma)\sigma \). With probability \( (1 - \gamma) \) the buyer will be unlucky obtaining the actual match value \( s - \sigma \) leading to a taste-loss with respect to the reference point of size \( \eta\lambda(s + (\gamma - \frac{1}{2})\sigma - (s - \frac{\sigma}{2})) = \eta\lambda\gamma\sigma \). Hence the consumer’s expected utility from buying his favorite product in period 1 is

\[
U_{1,B}(\sigma, B) = s + \gamma - \frac{1}{2})\sigma - (p_B - z_B) - \lambda\eta(1 - \gamma)\sigma.
\] (1)

The consumer’s expected utility from purchasing the cheaper but non-favored product \( A \) in period 1 can be calculated analogously and is given by

\[
U_{1,A}(\sigma, B) = s - (\gamma - \frac{1}{2})\sigma - (p_A - z_A) - \lambda\gamma(1 - \gamma)\sigma.
\] (2)

In period 1, a \( B \)-type consumer obtains a higher utility from buying his favorite product \( B \) rather than the cheaper product \( A \) unless his choosiness \( \sigma \) is sufficiently low. More precisely, \( B \)-types prefer purchasing \( B \) over purchasing \( A \) in period 1 if and only if \( U_{1,B}(\sigma, B) > U_{1,A}(\sigma, B) \Leftrightarrow \sigma \geq \bar{\sigma} \) where

\[
\bar{\sigma} \equiv \frac{p_B - p_A + z_A - z_B}{2\gamma - 1}.
\] (3)

Note that loss aversion has no effect on the buyer’s choice between product \( A \) and \( B \) in period 1. This is because the consumer’s choice between product \( A \) and product \( B \) and the formation of his expectations are based on the same information.

In period 2, the consumer’s choice between product \( A \) and product \( B \) is straightforward. This is because, any consumer who waited until period 2 must have done so in order to
purchase his preferred product. If the consumer’s purchase in period 2 was independent of his new information (i.e. the identity of his preferred product) then he could have made his purchase at a discount already in period 1. In other words, paying for additional information by forfeit of a discount makes sense only when the information is actually employed. Hence, in period 2 all consumers will purchase their preferred product.

In summary, we can therefore note that, independently of the time of purchase, the consumer’s choice between buying product A or buying product B is unaffected by his loss aversion. However, as we will see in the following subsection, loss aversion does have an effect on the consumer’s timing of purchase, i.e. his choice between an uninformed and an informed purchase.

3.2 Timing of purchase

Consider a consumer’s choice between purchasing his favorite product in advance or postponing the purchase until the identity of his preferred product has become known. For an A-type consumer with choosiness $\sigma$ the expected payoff from purchasing his favorite product A in advance can be derived in analogy to (1) and is given by

$$U_{1,A}(\sigma, A) = s + (\gamma - \frac{1}{2})\gamma - (p_A - z_A) - l_1\gamma(1 - \gamma)\sigma.$$  (4)

Note that in the match value dimension an advance purchase consists of the favorite product’s expected valuation minus a term that accounts for the consumer’s potential gains and losses from this expectation.

If the consumer postpones his purchase until period 2 he will secure the purchase of his preferred product, which guarantees a consumption value of size $s + \frac{\sigma}{2}$. However, when $p_A \neq p_B$, the consumer faces uncertainty about the price he will have to pay because ex ante the identity of his preferred product is unknown. Hence, the consumer’s (dis-)utility in the money dimension consists of the price he expects to pay for his preferred, yet unknown, product, minus a term accounting for the possible gains and losses from paying less or more than expected. More precisely, the consumer expects to pay $\gamma p_A + (1 - \gamma)p_B$ but given $p_A \leq p_B$, he will experience a money-gain $\eta[\gamma p_A + (1 - \gamma)p_B - p_A] = \eta(1 - \gamma)(p_B - p_A)$ with probability $\gamma$ and a money-loss $\eta\lambda[p_B - [\gamma p_A + (1 - \gamma)p_B]] = \eta\lambda(\gamma(p_B - p_A)$ with probability
1 − γ. Hence the consumer’s expected utility from postponing his purchase is given by

\[ U_2(\sigma, A) = s + \sigma \gamma - (1 - \gamma)p_B - l_m \gamma (1 - \gamma)(p_B - p_A). \]  

(5)

A consumer with favorite A will postpone his purchase until period 2 if and only if his choosiness is sufficiently high, i.e. if \( \sigma \) is such that \( U_2(\sigma, A) > U_1,A(\sigma, A) \) or equivalently \( \sigma > \sigma_{WA} \)

\[ \sigma_{WA} \equiv \frac{(1 - \gamma)(p_B - p_A) + z_A}{(1 - \gamma)(1 + l_\gamma)} + \frac{l_m \gamma}{1 + l_\gamma} (p_B - p_A). \]  

(6)

For B-type consumers an analog argument can be used to show that his expected utility from waiting is given by

\[ U_2(\sigma, B) = s + \sigma \gamma - (1 - \gamma)p_A - l_m \gamma (1 - \gamma)(p_B - p_A). \]  

(7)

A consumer with favorite B will postpone his purchase until period 2 if and only if \( U_2(\sigma, B) > U_{1,B}(\sigma, B) \) or equivalently \( \sigma > \sigma_{WB} \)

\[ \sigma_{WB} \equiv \frac{(1 - \gamma)(p_A - p_B) + z_B}{(1 - \gamma)(1 + l_\gamma)} + \frac{l_m \gamma}{1 + l_\gamma} (p_B - p_A). \]  

(8)

Note that loss aversion in money has the same negative effect on the two types’ willingness to purchase in advance. This is because both consumer types experience the same disutility from the uncertainty in period 2 payments induced by \( p_A \neq p_B \). Also note that the effects of loss aversion in taste are diametrically opposed. This is because a decrease in \( p_A \) raises the willingness of A-types to purchase in advance while lowering the willingness of B-types. Loss aversion in taste makes both types’ timing of purchase less sensitive with respect to the decrease in \( p_A \).

In the above derivation we have implicitly assumed that, when indifferent with respect to their timing of purchase, consumers would choose their product in accordance with their signal in period 1. This holds when first period prices are similar (which will be the case due to our assumption of symmetry), i.e. \( (p_A, z_A) \approx (p_B, z_B) \), because in this case \( \bar{\sigma} \approx 0 \). When \( (p_A, z_A) = (p_B, z_B) = (p, z) \) the fraction of consumers who purchase in advance is given by

\[ \sigma_W \equiv \frac{z}{(1 - \gamma)(1 + l_\gamma)}. \]  

(9)
Note that for $p_A = p_B$, only loss aversion in taste has a direct effect on advance selling. Loss aversion in taste reduces advance selling directly by lowering the consumers’ willingness to make an uninformed purchase. However, loss aversion (in both dimensions) also has an indirect effect on advance selling by changing the discounts that firms charge in equilibrium. It is the interaction of these two effects that renders the analysis of the influence of loss aversion on advance selling non-trivial.

4 Advance selling and allocative efficiency

In advance purchase settings where consumers have unit demands and valuations that are large enough for the market to be covered, the fraction of advance sales, $\sigma_W$, provides us with a measure of market performance. More specifically, in our setting, allocative efficiency is given by

$$V = 2s + \int_0^{\sigma_W} [\gamma\sigma - (1 - \gamma)\sigma]f(\sigma)d\sigma + \int_{\sigma_W}^1 \sigma f(\sigma)d\sigma. \quad (10)$$

$V$ measures the (expected) aggregate consumption value generated by the market’s allocation of the products $A$ and $B$ when consumers with choosiness $\sigma \in [0, \sigma_W]$ are induced to purchase in advance while consumers with choosiness $\sigma \in (\sigma_W, 1]$ guarantee the consumption of their preferred product by postponing their purchase. Note that, in the presence of loss aversion, allocative efficiency differs from welfare, in that it fails to account for the gain-loss term $-l(1 - \gamma)\sigma$ in the consumers’ (expected) utility from advance purchases. Independently of which measure of market performance is considered, market performance is strictly decreasing in the fraction of advance sales. For this reason, the fraction of advance sales, $\sigma_W$, is the focus of our analysis in this section.

Our goal is to understand how the relationship between advance selling ($\sigma_W$) and information ($\gamma$) is affected by the existence of consumer loss aversion ($l$). Understanding this relationship is important because, as can be seen from (10), information affects market performance not only directly (by reducing the likelihood of mismatches) but also indirectly through its influence on the inter-temporal allocation of sales. In Sections 4.1 and 4.2 we determine the market’s fraction of advance sales $\sigma_W$ separately for the cases of monopoly and
competition. In both cases, accounting for loss aversion leads to a systematic change in the way advance selling depends on information. In Section 4.3 we argue that, for a large family of distributions $f$, this change reverses our assessment (in terms of allocative efficiency) of policies that improve consumers’ information in advance purchase markets.

### 4.1 Monopoly

We start our analysis by considering a monopolistic market where products $A$ and $B$ are sold by the same firm. Due to the model’s symmetry, the monopolist will choose the same pricing strategy $(p^M, z^M)$ for his two products.\(^{10}\) Consumers with choosiness $\sigma \leq \sigma^M_W = \frac{z^M}{(1-\gamma)(\gamma+1)}$ will purchase their favorite product in advance whereas consumers with choosiness $\sigma > \sigma^M_W$ will postpone their purchase until the consumption period. From (1) and (4) it follows that a consumer’s utility from purchasing his preferred product in advance is linear in his type $\sigma$. For

$$\gamma - \frac{1}{2} > l(1-\gamma) \Leftrightarrow \gamma > \gamma^M(l) \equiv \frac{l - 1 + \sqrt{1 + l^2}}{2l},$$

advances-purchase utility is increasing in $\sigma$ and hence minimized at $\sigma = 0$. For $\gamma < \gamma^M(l)$, advance-purchase utility is decreasing and minimized at $\sigma = \sigma^M_W$. Given our assumption of a covered market, all consumers who purchase in advance must obtain non-zero utility. To extract the maximum rent, the monopolist must make the consumer with the lowest advance-purchase utility indifferent between buying and not buying.\(^{11}\) Hence it must hold that

$$p^M - z^M = \begin{cases} s - [l(1-\gamma) - \gamma + \frac{1}{2}]\sigma^M_W & \text{if } \gamma < \gamma^M(l) \\ s & \text{if } \gamma \geq \gamma^M(l) \end{cases}$$

and substitution into the monopolist’s profits

$$\Pi^M = 2((p^M - z^M)F(\sigma^M_W) + p^M[1 - F(\sigma^M_W)])$$

\(^{10}\) One may suspect that setting asymmetric (regular) prices, $p_A \neq p_B$, could be optimal in spite of the model’s symmetry. When consumers are loss averse, setting $p_A \neq p_B$ renders purchasing in advance more attractive, i.e. asymmetric prices may serve as a substitute for a discount. Yet, as we show in the proof of Proposition 1, this presumption is incorrect, i.e. the monopolist’s profit maximizing pricing strategy is symmetric.

\(^{11}\) Note that a consumers’ expected utility from purchasing in period 2 is always increasing in his type $\sigma$. Hence, participation of types in $[0, \sigma^M_W]$ guarantees participation of types in $(\sigma^M_W, 1]$.
reveals that the monopolist’s problem can be reduced to the choice of $\sigma^M_W$ that maximizes
\[
\frac{\Pi^M}{2} = \begin{cases} 
    s - [l\gamma(1 - \gamma) - \gamma + \frac{1}{2}]\sigma^M_W + (1 - \gamma)(ly + 1)\sigma^M_W(1 - F(\sigma^M_W)) & \text{if } \gamma < \gamma^M(l) \\
    s + (1 - \gamma)(ly + 1)\sigma^M_W(1 - F(\sigma^M_W)) & \text{if } \gamma \geq \gamma^M(l).
\end{cases}
\] (14)

It is immediate that for $\gamma \geq \gamma^M(l)$ the solution to this maximization problem is independent of the consumers’ degree of loss aversion. Define this solution as
\[
\sigma^0_W \equiv \arg \max_{\sigma_W \in [0,1]} \sigma_W(1 - F(\sigma_W)).
\] (15)

Our assumption that $f$ has an increasing hazard rate guarantees that $\sigma^0_W$ is uniquely defined and $\sigma^0_W \in (0, 1)$. For $\gamma < \gamma^M(l)$, the need to compensate type $\sigma^M_W$ for his loss aversion through a price reduction of size $[l\gamma(1 - \gamma) - \gamma + \frac{1}{2}]\sigma^M_W$ lowers the monopolist’s incentive to sell in advance. In the Appendix we prove the following:

**Proposition 1 (Advance Selling - Monopoly).** For every $l \in (0, 2)$ there exists $\gamma^M(l) \in (\frac{1}{2}, 1)$ such that the following holds. For $\gamma \geq \gamma^M(l)$ the monopoly allocation of sales is independent of $l$ and identical to the one without loss aversion given by $\sigma^M_W = \sigma^0_W$. For $\gamma < \gamma^M(l)$, loss aversion reduces advance selling, i.e. $\sigma^M_W < \sigma^0_W$.

In the absence of loss aversion, the monopoly allocation of sales is independent of the consumers’ information. This changes when loss aversion is taken into account. As can be seen from the left hand panel of Figure 2, loss aversion reduces monopolistic advance selling when consumer’s are relatively uninformed about their preferences. The figure depicts the case of a uniform distribution $f$ of consumer types for which
\[
\sigma^M_W = \begin{cases} 
    \frac{1}{4(l - \gamma)(\gamma + 1)} & \text{if } \gamma < \gamma^M(l) \\
    \frac{1}{2} & \text{if } \gamma \geq \gamma^M(l).
\end{cases}
\] (16)

With a uniform distribution, or more generally for $f$ non-increasing (see proof of Proposition 1), monopolistic advance selling becomes an increasing function of the quality of the consumers’ information.

Investigating a firm’s incentive to provide consumers with information about its product’s characteristics, Lewis and Sappington (1994), Bar Isaac et al. (2010), and Gill and Sgroi (2012) show that a monopolistic seller may often find it optimal to provide consumers with either full information or none. The limiting cases of an uninformed ($\gamma \to \frac{1}{2}$) and an informed market ($\gamma \to 1$) therefore deserve some special attention:
Corollary 1. Without consumer loss aversion, the monopolist’s inter-temporal allocation of sales is independent of the quality of consumers’ information. When consumers are loss averse \( l > 0 \), a monopolist practices more advance selling in an informed market than in an uninformed market, i.e. \( \lim_{\gamma \to 1} \sigma^M_W > \lim_{\gamma \to 1} \frac{1}{2} \sigma^M_W \).

Proposition 1 and its corollary show that, when consumer loss aversion is accounted for, a monopolist’s incentive to sell in advance depends on the quality of the consumer’s information. For a monopolistic market, policies that improve the consumers’ information during the advance purchase period can then have the adverse effect of increasing the fraction of consumers who purchase in advance. As we will argue next, a very different picture emerges in the presence of competition.

4.2 Competition

Consider the case where products A and B are offered by two competing firms. By offering an advance purchase discount a firm is able to secure the demand of a consumer who might
become the rival’s customer after learning his true preferences. If firm A chooses a pricing policy \((p_A, z_A)\) such that \(p_A \leq p_B\) and \(z_A \geq z_B\) then its profit is given by

\[\Pi_A = (p_A - z_A)[F(\sigma_{WA}) + F(\tilde{\sigma})] + p_A\gamma[1 - F(\sigma_{WA})] + (1 - \gamma)[1 - F(\sigma_{WB})].\] (17)

Here the thresholds \(\tilde{\sigma}, \sigma_{WA},\) and \(\sigma_{WB}\) are given by (3), (6), and (8) respectively, and we have used the fact that for small deviations from a symmetric equilibrium pricing policy it must hold that \(\tilde{\sigma} < \sigma_{WB}\). The firm’s revenue has four parts. In period 1, firm A sells at discounted price \(p_A - z_A\) to A-types who are sufficiently unchoosy to purchase in advance and to B-types who are sufficiently unchoosy to be attracted by A in spite of favoring (the more expensive product) B. In period 2, firm A sells at regular price \(p_A\) to those consumers who were too choosy to buy in advance. Firm A sells with probability \(\gamma\) to consumers whose favorite was A and with probability \(1 - \gamma\) to consumers whose favorite was B.

Taking derivatives of (17) with respect to \(p_A\) and \(z_A\) and substituting \(p_A = p_B = p^*\) and \(z_A = z_B = z^*\) leads the following two first order conditions that have to be satisfied by any (symmetric) equilibrium:

\[0 = -1 + p^*[\frac{f(0)}{2\gamma - 1} + \frac{2(1 - \gamma)f(\sigma_{WA}^*)}{1 + l\gamma}] - z^*[\frac{f(0)}{2\gamma - 1} + f(\sigma_{WA}^*)]\] (18)

\[0 = \frac{p^* - z^*}{2\gamma - 1}f(0) - F(\sigma_{WA}^*) - \gamma p^* - p^* + z^*\]
\(= \frac{\gamma p^* + p^*}{1 - \gamma}(1 + l\gamma)f(\sigma_{WA}^*).\) (19)

Using the relation \(z^* = (1 - \gamma)(1 + l\gamma)\sigma_{WA}^*\) these two equations can be combined leading to the following (implicit) equation for the equilibrium fraction of advance sales \(\sigma_{WA}^*:\)

\[0 = 1 + (1 - \gamma)(1 + l\gamma)\sigma_{WA}^*[\frac{f(0)}{2\gamma - 1} + f(\sigma_{WA}^*)]\] (20)

\[- F(\sigma_{WA}^*) + \sigma_{WA}^*f(\sigma_{WA}^*) + \sigma_{WA}^*\frac{(1 - \gamma)(1 + l\gamma)}{2\gamma - 1}f(0)]\quad 1 - \gamma(4\gamma - 2)f(\sigma_{WA}^*) + (1 + l\gamma)f(0) = (2\gamma - 1)f(\sigma_{WA}^*) + (1 + l\gamma)f(0).

For a uniform distribution \(f\) of consumer types the solution \(\sigma_{WA}^*\) to this equation can be obtained in closed form as

\[\sigma_{WA}^* = \frac{\gamma(l + 2)}{\gamma(3 + 4(1 - \gamma)) - 1 + \gamma[2 - (1 - \gamma)\gamma(l + 4)]}.\] (21)

We depict it in the right hand panel of Figure 2, both for the case of standard preferences \((l = 0)\) and loss aversion \((l > 0)\). As can be seen from the figure, loss aversion increases
advance selling when consumers are relatively uninformed but decreases advance selling when the consumers’ information is relatively precise. The following proposition shows that this insight is not restricted to a uniform distribution of consumer-types but holds more generally:

**Proposition 2** (Advance Selling - Competition). *In the presence of competition, consumer loss aversion increases advance selling in uninformed markets but reduces advance selling in informed markets, i.e. \( \frac{\partial}{\partial l} \lim_{\gamma \to 1} \sigma_w^* > 0 \) and \( \frac{\partial}{\partial l} \lim_{\gamma \to 1} \sigma_w^* \leq 0 \), with strict inequality for all \( l > \frac{1}{f(0)} - 1 \).*

The intuition for this result is as follows. In the presence of competition, firms engage in inter-temporal business stealing. Firms benefit from an advance purchase discount because it secures them the demand from those consumers who favor their product in the advance purchase market but would have turned out to prefer their rival’s product in the spot market. An increase in loss aversion raises the discount necessary to induce advance purchases. In a market with low uncertainty, the benefits from inter-temporal business stealing are relatively low because only a small fraction of consumers experiences a preference reversal. Hence firms will increase their discounts only little and the total effect is a reduction in advance selling. In contrast, Proposition 2 shows that in markets with high uncertainty, loss aversion raises the fraction of consumers who are served in advance. When uncertainty is high, inter-temporal business stealing is very beneficial as consumers are quite likely to reverse their preferences. In this case, firms have a strong incentive to engage in inter-temporal business stealing and the increase in discounts more than offsets the consumers’ raised reluctance to purchase in advance.

An immediate consequence of Proposition 2 is that loss aversion changes the way in which advance-selling depends on the quality of consumers’ information. In particular, Proposition 2 has the following implication:

**Corollary 2.** *Without consumer loss aversion, competing firms practice more advance selling in an informed market than in an uninformed market, i.e. for \( l = 0 \) it holds that \( \lim_{\gamma \to 1} \sigma_w^* \geq \lim_{\gamma \to 1} \sigma_w^* \). When consumers are loss averse, this conclusion can become reversed. In particular, if \( f(0) > \frac{1}{3} \) then there exists \( l \in (0, 2) \) such that for all \( l \in (l, 2) \),
\[
\lim_{\gamma \to 1} \sigma^*_{\omega} < \lim_{\gamma \to 1} \sigma^*_{\omega}, \text{ i.e. improving consumers' information reduces inter-temporal business stealing.}
\]

Note that the weak inequality in Corollary 2 becomes strict under very mild conditions on the distribution of consumer types \( f \). In particular, in the proof of Corollary 2 we show that, in the absence of loss aversion, there is strictly more advance selling in an informed market than in an uninformed market when the hazard rate of \( f \) is strictly increasing.

Proposition 2 and its corollary show that loss aversion introduces a systematic change in the way how firms’ incentive to engage in inter-temporal business stealing depends on the consumers’ information. With standard preferences, we are lead to the conclusion that improving consumer information intensifies inter-temporal business stealing. Accounting for loss aversion shows that this must not be the case. Instead, improving consumer information can be beneficial, because it mitigates inter-temporal business stealing.

### 4.3 The effect of information on allocative efficiency

In Sections 4.1 and 4.2 we have shown that accounting for loss aversion changes the way advance selling depends on information, both under monopoly and under competition. Without loss aversion, improving the consumers’ information increases the number of uninformed purchases in the presence of competition but has no effect under monopoly. In contrast, when we account for loss aversion, information increases advance selling under monopoly but may reduce advance selling in the presence of competition. Equipped with these new insights about the relation between information and advance selling we are now ready to consider the overall effect on allocative efficiency. We show that, for a large family of distributions, accounting for consumer loss aversion reverses our assessment of policies that improve consumer information.

For this purpose, suppose that the distribution of consumer types takes the form of a truncated normal, parametrized by the normal distribution’s mean \( \mu \in \mathbb{R} \) and variance \( \nu^2 > 0 \).
More precisely, for $\sigma \in [0, 1]$ define

$$f_{\mu, \nu}(\sigma) = \frac{\phi\left(\sigma - \frac{\mu}{\nu}\right)}{\nu[\Phi\left(\frac{\nu}{\nu} - \frac{\mu}{\nu}\right) - \Phi\left(\frac{-\mu}{\nu}\right)]}$$

(22)

with $\phi(\sigma) = \frac{1}{\sqrt{2\pi}} \exp \left(-\frac{\sigma^2}{2}\right)$ and $\Phi(\sigma) = \int_{-\infty}^{\sigma} \phi(\tilde{\sigma}) d\tilde{\sigma}$ denoting the probability density function and the cumulative distribution function of the standard normal distribution. The family of truncated normals covers increasing, decreasing, as well as single-peaked densities of consumer-choosiness. Determining the fraction of advance sales for a truncated normal $f_{\mu, \nu}$ amounts to finding the (unique) root $\sigma_W \in (0, 1)$ of (20) for competition and of (30), resp. (31) in the proof of Proposition 1, for monopoly, after substitution of $f$ by (22). After determining $\sigma_W$ (numerically) we can use (10) to calculate allocative efficiency. Repeating this process for different values of $\gamma$ we are able to analyze the dependence of allocative efficiency on the quality of consumers’ information. In Figure 4 we depict an example that highlights how this dependence changes with loss aversion. The left hand panel depicts the case of a monopoly. Without loss aversion (solid), allocative efficiency is monotonically increasing with information. This changes when consumers are loss averse (dotted). In particular, there exists a range of $\gamma$ for which allocative efficiency is decreasing with information. In this range, the increase in the frequency of advance selling is strong enough to more than offset the reduced risk of a mismatch. Hence, for a monopoly, accounting for consumer loss aversion may change our assessment of information policies from positive to indeterminate.

The opposite happens in the case of competition depicted in the right hand panel of Figure 4. Without loss aversion, allocative efficiency first decreases and then increases with information. In contrast, when consumers are loss averse, allocative efficiency turns out to be monotonically increasing with information. Inter-temporal business stealing increases less or even decreases with information, so that the reduction in the risk of a mismatch dominates the indirect effect of a change in the frequency of advance selling. Hence, for the case of competition, accounting for loss aversion may change our assessment of information policies from indeterminate to positive.

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12The truncated normal distribution has an increasing hazard rate (and thus satisfies our assumptions) because the normal distribution is log-concave, truncation keeps log-concavity intact, and log-concavity is sufficient for an increasing hazard rate (Bagnoli and Bergstrom, 2005).
Figure 2: **Allocative efficiency.** $V$ as a function of the quality of consumers’ information $\gamma$. The left hand panel shows monopoly, the right hand panel shows competition. Solid curves depict standard preferences ($l = 0$), dotted curves depict loss aversion ($l = 1.5$). In this example, the distribution of consumers’ choosiness is given by the truncated normal $f_{\mu, \nu}$ defined in (22) with parameters $\mu = 0.2$ and $\nu = 0.4$.

The influence of consumer loss aversion on the desirability of information in advance purchase markets is not restricted to this particular example but turns out to be rather generic. More specifically, as can be seen from Figure 5, loss aversion changes the effect of information on allocative efficiency, from increasing to U-shaped for monopoly and from U-shaped to monotonic under competition, for a large set of distributions within the family of truncated normals. We can therefore conclude that accounting for consumer loss aversion and market structure constitute important elements in our assessment of information policy in advance purchase markets.

## 5 Pricing and consumer surplus

In this section we consider the effect of consumer loss aversion on prices. Our ultimate goal is to understand how the relation between information and consumer surplus is affected by the presence of consumer loss aversion. In line with the existing literature on this issue, we assume for the remainder that the distribution of consumer-types is uniform.
Figure 3: **Value of information.** Each figure depicts whether, for a particular truncated normal distribution $f_{\mu,\nu}$ of consumer choosiness, allocative efficiency is monotonically increasing (+) or U-shaped (blank) in the quality of consumers’ information $\gamma$. For monopoly (left hand panels) the switch from no loss aversion ($l = 0$, upper panel) to loss aversion ($l = 1$, lower panel) reveals that information can be detrimental. For competition (right hand panels) the same switch has a very different effect; it reveals that information is actually more beneficial than when loss aversion is failed to be accounted for.

### 5.1 Monopoly

For $f$ uniform, we can use (9), (12), and (16) to determine the monopolist’s profit maximizing prices in closed form:

$$p^M - z^M = \begin{cases} s - \frac{ly(1-\gamma) - \gamma + \frac{1}{2}}{4(1-\gamma)(1+ly)} & \text{if } \gamma < \gamma^M(l) \\ s & \text{if } \gamma \geq \gamma^M(l) \end{cases}$$  \hspace{1cm} (23)

$$p^M = \begin{cases} s + \frac{1}{4} - \frac{ly(1-\gamma) - \gamma + \frac{1}{2}}{4(1-\gamma)(1+ly)} & \text{if } \gamma < \gamma^M(l) \\ s + \frac{1}{2}(1-2\gamma)(1+ly) & \text{if } \gamma \geq \gamma^M(l) \end{cases}$$  \hspace{1cm} (24)
where $\gamma^M(l)$ is as defined in (11).

**Proposition 3** (Pricing - Monopoly). Suppose $f$ is uniform. The monopolist’s profit maximizing prices are given by (23) and (24). For $\gamma \geq \gamma^M(l)$, loss aversion increases regular price but has no effect on discounted price, i.e. $\frac{\partial p^M}{\partial l} > 0$ and $\frac{\partial p^M - z^M}{\partial l} = 0$. For $\gamma < \gamma^M(l)$, loss aversion has a negative effect on both prices i.e. $\frac{\partial p^M - z^M}{\partial l} < 0$ and $\frac{\partial p^M}{\partial l} < 0$.

To understand this result, recall that for small $\gamma$, the monopolist chooses the advance purchase price $p^M - z^M$ to make the consumer with type $\sigma_w^M$ indifferent between participating and not participating in the market. Loss aversion lowers the consumers’ expected utility from participating in the market. In order to compensate consumers’ for their increase in gain/loss disutility the monopolist reduces his prices $p^M - z^M$ and $p^M$ (by equal amounts). For large enough $\gamma$, our analysis in the previous section has shown that the monopolist’s profits are maximized when he sells to exactly half of the consumers in advance. Loss aversion increases the discount that is necessary to achieve this and, given that advance purchase prices $p^M - z^M$ are set equal to $s$, therefore leads to an increase in the regular price $p^M$.

Accounting for loss aversion changes the way in which monopoly pricing depends on information, as summarized by the following corollary:

**Corollary 3.** Suppose $f$ is uniform. Without consumer loss aversion, monopoly prices $p^M - z^M$ and $p^M$ are constant and decreasing in the quality of consumers’ information $\gamma$, respectively. When consumers are loss averse, there exists a range $(\frac{1}{2}, \gamma^M(l))$ in which monopoly prices are increasing in $\gamma$.

The effect of information on monopoly pricing is depicted in the left hand panel of Figure 6. Accounting for consumer loss aversion reveals that improving consumers’ information may lead to an increase in monopoly prices. This insight is important because it implies that, in monopolistic advance purchase markets, information may have an adverse effect not only on (allocative) efficiency but also on consumer surplus. We will come back to this issue in Section 5.3.
Figure 4: Pricing. Regular price $p$ and discounted price $p - z$ as a function of the quality of consumers’ information $\gamma$ for $f$ uniform and $s = 1$. The left hand panel shows monopoly, the right hand panel shows the case of competition. Solid curves depict standard preferences ($l = 0$), dashed curves depict loss aversion ($l = 1.5$).

5.2 Competition

For oligopolistic markets without an advance purchase option it has been found that whether loss aversion has a pro-competitive or an anti-competitive effect depends on the relative strength of loss aversion in the money and the taste dimensions. In our analysis of duopoly pricing below, we therefore distinguish between these two dimensions of loss aversion by allowing $l_m$ to differ from $l_t$. We derive the firms’ symmetric equilibrium pricing policy $(p^*, z^*)$ in two steps. First, we take the price level $p_A = p_B = p$ as exogenous and determine the unique equilibrium candidate discount $z^*(p)$. Second, we take discounts $z_A = z_B = z$ to be exogenous and determine the unique equilibrium candidate price level $p^*(z)$. The candidate equilibrium is then given by the unique solution to the system of equations $p^*(z^*) = p^*$ and $z^*(p^*) = z^*$. Decomposing the equilibrium analysis into these two steps improves our understanding of the difference between competition in price levels and competition in discounts in advance purchase markets.
Step 1: Competition in discounts. Suppose that the price level is exogenous, \( p_A = p_B = p \). Which discount \( z^* \) would firms offer in a (symmetric) equilibrium? Taking \( f \) to be uniform, the first order condition in (19) simplifies to:

\[
\frac{p - z^*}{2\gamma - 1} - \frac{z^*}{(1 - \gamma)(1 + l_\gamma)} = 0.
\]

This condition shows the effects of an increase in firm A’s discount. It balances the gain from attracting B-types to purchase product A in advance, with the losses from granting a higher discount to a fraction \( \sigma W \) of firm A’s customers, and from attracting additional A-types to purchase from firm A in advance rather than on the spot.

Note that, with the price-level exogenous, loss aversion in money has no influence on the firms’ choice of discounts. This is because for \( p_A = p_B = p \) second period payments become deterministic and loss aversion in money has no bite. Also note from (25) that, in equilibrium, discounts must be chosen such that \( \gamma p - p + 2z^* > 0 \). This implies that loss aversion in the taste dimension reduces the losses from granting a higher discount, giving firms’ a stronger incentive to increase their discounts. Hence we have shown the following result:

**Lemma 1.** Let \( f \) be uniform and suppose that the price-level is exogenous, \( p_A = p_B = p \). Loss aversion in money has no effect on discounts, i.e. \( z^*(p) \) is independent of \( l_m \). In contrast, loss aversion in taste intensifies competition in discounts, i.e. \( z^*(p) \) is increasing in \( l_t \).

Step 2: Competition in price levels. Suppose that discounts are exogenous, \( z_A = z_B = z \). Which price-level \( p^*(z) \) would firms choose in a (symmetric) equilibrium? Taking \( f \) to be uniform, the first order condition in (18) simplifies to:

\[
-1 + p^*[\frac{1}{2\gamma - 1} + \frac{2(1 - \gamma)}{1 + l_\gamma}] - z[\frac{1}{2\gamma - 1} + \frac{1 + l_m\gamma}{1 + l_\gamma}] = 0.
\]

This condition shows the effect of a price-decrease on firm A’s profit. Apart from the obvious loss from selling to a mass one of consumers at a lower price, a price-cut has two effects; it increases the firm’s total number of customers, and it raises the number of discounts the firm has to grant. From (26), the effect of loss aversion in money is straightforward.\(^{13}\)

\(^{13}\)The effect of loss aversion in taste depends on the size of the exogenous discount \( z \). For details see the proof of Lemma 2 in the Appendix.
Loss aversion in money increases the firm’s profit-reduction from having to grant a larger number of discounts. This is because loss aversion in money raises the consumers’ disutility from the second-period price-uncertainty induced by the firm’s deviation from equilibrium, making consumers more attracted by the discount.

**Lemma 2.** Let \( f \) be uniform and suppose that discounts are exogenous, \( z_A = z_B = z \). Loss aversion in money mitigates competition in price-levels, i.e. \( p^*(z) \) is increasing in \( l_m \).

**Equilibrium.** For \( f \) uniform, the unique candidate for a symmetric pure-strategy equilibrium can be obtained in closed form from solving \( z^*(p^*) = z^* \) and \( p^*(z^*) = p^* \) simultaneously, with \( z^*(p) \) and \( p^*(z) \) denoting the solutions of (25) and (26) respectively. We obtain:

\[
z^* = \frac{(1 - \gamma)\gamma(l_t + 2)(1 + l_t\gamma)}{\gamma [7 + 2 l_t - (1 - \gamma)\gamma(l_m + 2)l_t - 2\gamma((1 - \gamma)l_m + 2)] - 1}
\]

\[
p^* = \frac{(1 + l_t\gamma)(1 - \gamma)\gamma l_t + 3\gamma - 1}{\gamma [7 + 2 l_t - (1 - \gamma)\gamma(l_m + 2)l_t - 2\gamma((1 - \gamma)l_m + 2)] - 1}.
\]

Uniqueness is immediate from the linearity of \( z^*(p) \) and \( p^*(z) \). Existence has been shown by Möller and Watanabe (2016) for the case \( l_t = l_m = 0 \). By continuity, existence is thus guaranteed as long as the consumers’ degree of loss aversion is sufficiently small. Based on the insights of Lemmas 1 and 2, in the Appendix we prove the following:

**Proposition 4 (Pricing - Competition).** Let \( f \) be uniform. Loss aversion has an anti-competitive effect on prices but a pro-competitive effect on advance purchase discounts, i.e. \( p^*, p^* - z^* \), and \( z^* \) are monotonically increasing in \( l \).

Proposition 4 describes the effect of loss aversion on competing firms’ choice of pricing policy. In light of Karle and Peitz’s (2014) finding that loss aversion acts anti-competitively

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14In our setting, the following argument shows that in any symmetric pure-strategy equilibrium, firms must offer advance purchase discounts, \( z^* > 0 \). Suppose instead that \( z^* = 0 \) and firm A deviates by offering a small discount \( z_A > 0 \). Firm A’s discount will attract some of the least choosy consumers to purchase from firm A in advance at a price \( p^* - z_A \). Consumers who become attracted by the discount would have become firm A’s customers in period 2 at price \( p^* \) with probability \( \gamma \) (for A-types) or \( 1 - \gamma \) (for B-types) respectively. Choosing \( z_A \) sufficiently small to satisfy \( p^* - z_A > \gamma p^* \), we have identified a profitable deviation. Hence in equilibrium it must hold that \( z^* > 0 \).

15For larger \( l \), showing existence is a tedious but straightforward exercise analogous the lines of Möller and Watanabe (2016).
in the taste-dimension but pro-competitively in the money-dimension, it is important to note that Proposition 4 holds independently of whether loss aversion acts only in taste \((l = l_t > 0 = l_m)\) or only in money \((l = l_m > 0 = l_t)\) or receives equal weight in both dimensions \((l = l_t = l_m > 0)\). Surprisingly, in an advance purchase setting, prices turn out to be increasing in loss aversion, independently of its dimension.

Accounting for loss aversion changes the way in which equilibrium prices depend on information, as summarized by the following corollary:

**Corollary 4.** Suppose \(f\) is uniform. Without consumer loss aversion, equilibrium prices are increasing in the quality of consumers’ information \(\gamma\). For \(l > 1\) there exists a range \((\gamma^*, 1)\) where spot prices \(p^*\) are decreasing in \(\gamma\).

The effect of information on equilibrium prices can be seen in the right hand panel of Figure 6. The reason why advance purchase prices \(p^* - z^*\) are increasing in \(\gamma\) is that when consumers are relatively well informed, they differentiate strongly between products (already in the advance purchase market). Product differentiation mitigates competition, leading to higher prices. As shown by Proposition 4, loss aversion acts pro-competitively on discounts, i.e. it widens the gap between advance purchase prices \(p^* - z^*\) and spot prices \(p^*\). As this gap is largest for \(\gamma \to \frac{1}{2}\) but vanishes for \(\gamma \to 1\), loss aversion can make spot prices \(p^*\) decrease with information when consumers are relatively well informed.

### 5.3 The effect of information on consumer surplus

How does information affect consumer surplus in advance purchase markets? In order to answer this question consider (expected) aggregate consumer surplus:

\[
CS = \int_0^{\sigma_w} [s + (\gamma - \frac{1}{2})\sigma - l\gamma(1 - \gamma) - (p - z)]d\sigma + \int_{\sigma_w}^1 [s + \frac{\sigma}{2} - p]d\sigma. \tag{29}
\]

Consumer surplus consists of two parts; the surplus of consumers who purchase their favorite product in advance at price \(p - z\), and the surplus of consumers who purchase their preferred product on the spot at price \(p\). Note that, as common in the literature, the consumers’ gain-loss utility is included in our calculation of consumer surplus.
Improving information has the obvious effect of enabling consumers to make better purchase decisions. When, in addition, information decreases prices the overall effect on consumer surplus is unambiguous. In this case, information increases consumer surplus. In contrast, when prices are increasing with information, the overall effect is unclear.

In Figure 7, we depict consumer surplus, both for the case of monopoly and the case of competition. In the absence of loss aversion, information affects consumer surplus in opposite directions, depending on market structure. For a monopoly, prices are (weakly) decreasing with information (Corollary 3) and hence, consumer surplus is increasing. Under competition, prices are increasing with information and the price increase turns out to be sufficiently strong to make consumer surplus a decreasing function of information. Account-

![Figure 5: Consumer surplus. CS as a function of the quality of consumers’ information γ for f uniform. The left hand panel shows monopoly, the right hand panel shows the case of competition. Solid curves depict standard preferences (l = 0), dashed curves depict loss aversion (l = 1.5).](image)

...ing for consumer loss aversion reveals that information can be detrimental for consumers not only in oligopolistic but also in monopolistic markets. While in an oligopolistic market, information reduces consumer surplus because it mitigates competition, under monopoly information is detrimental (for relatively small γ) because it decreases the gain/loss disutility
of the “marginal” type (the one indifferent between buying and not buying), allowing the monopolist to extract a higher rent from all consumers.

6 Conclusion

Contributing to the emerging literature on Behavioral Industrial Organization, this paper has introduced expectation-based loss aversion à la Kőzegi and Rabin (2006, 2007) into an advance purchase setting. Our starting point was the observation that in advance purchase markets this particular form of non-standard risk preferences can be expected to be especially relevant, because consumers are required to form expectations about prices and product match in order to decide on their timing of purchase. The paper’s main results are concerned with the relation between information and market performance (fraction of uninformed purchases, allocative efficiency, prices, consumer surplus), with an emphasis on how this relation changes when consumer loss aversion is accounted for. The paper’s analysis covers both the case of monopoly and the case of competition.

For a monopoly, we find that, in the absence of loss aversion, improving consumers’ information has no effect on advance selling and thus a positive effect on allocative efficiency. Prices decrease and consumer surplus increases with information. Accounting for loss aversion reveals that improving information may increase advance selling and thus be detrimental for allocative efficiency. Moreover, prices may increase with information, leading to a reduction in consumer surplus.

For a duopoly, we find that, without consumer loss aversion, better information leads to more advance selling, making the overall effect on allocative efficiency indeterminate. In contrast, when consumers are (sufficiently) loss averse, information will reduce advance selling, and hence be unambiguously beneficial for allocative efficiency. Regarding prices and consumer surplus, loss aversion produces no (qualitative) change in their dependence on information.

In short, we can therefore conclude that accounting for consumer loss aversion affects our assessment of the desirability of information in advance purchase markets; negatively for a monopoly but positively in the presence of competition. More generally, our results
demonstrate that accounting for consumers’ risk preferences in markets where consumer decisions are made (at least partly) under uncertainty, may have a first-order impact on the prospective outcome of regulation.

Appendix: Proofs

Proof of Proposition 1. Consider the first order condition corresponding to the maximization program in (15):

$$\frac{1 - F(\sigma_W)}{f(\sigma_W)} - \sigma_W = 0.$$  
(30)

Because $f$ has an increasing hazard rate the LHS is strictly decreasing in $\sigma_W$. It is positive for $\sigma_W = 0$ and negative for $\sigma_W = 1$. By continuity there exists a unique solution, i.e. $\sigma^0_W$ is well-defined. For $\gamma \geq \gamma^M(l)$, $\sigma^0_W$ maximizes the monopolist’s payoff, i.e. $\sigma^M_W = \sigma^0_W$ is independent of $l$. In the following, consider the remaining case where $\gamma < \gamma^M(l)$. The monopolist’s problem implies the first order condition

$$\frac{1 - F(\sigma_W)}{f(\sigma_W)} - \sigma_W - \frac{l\gamma(1 - \gamma) - \gamma + \frac{1}{2}}{(1 - \gamma)(l + 1)f(\sigma_W)} = 0.$$  
(31)

The LHS of (31) is strictly smaller than the LHS of (30) for all $\sigma_W$. This implies that for $\gamma < \gamma^M(l)$ the profit-maximizing allocation of sales must be such that $\sigma^M_W < \sigma^0_W$. If $f$ is non-increasing then the LHS of (31) is strictly decreasing in $\sigma_W$. Because the last fraction in (31) can be written as $\frac{1}{f(\sigma_W)} - \frac{l\gamma(1 - \gamma)}{2f(\sigma_W)(1 - \gamma)l(1 + l)}$ and because $(1 - \gamma)(1 + l)$ is decreasing in $\gamma$ but increasing in $l$ it also holds that the LHS of (31) is increasing in $\gamma$ but decreasing in $l$. Hence, for $f$ non-increasing, the Implicit Function Theorem implies that (31) has a unique solution, $\sigma^M_W$, and that $\sigma^M_W$ is increasing in $\gamma$ but decreasing in $l$. When $f$ is uniform, it follows from (31) that $\sigma^M_W = \frac{1}{\kappa(1 - \gamma\gamma l + 1)}$.

Asymmetric pricing: As mentioned in footnote 10, we show next that the monopolist has no incentive to set asymmetric regular prices. For this purpose, suppose that $p_B = p_A + \Delta p$ with $\Delta p \geq 0$ and $z_A \geq z_B$. Then, the monopolist’s profit is given by

$$\Pi^M = (p_A - z_A)[F(\sigma_{WA}) + F(\tilde{\sigma})] + p_A[\gamma[1 - F(\sigma_{WA})] + (1 - \gamma)[1 - F(\sigma_{WB})]] + (p_B - z_B)[F(\sigma_{WB}) - F(\tilde{\sigma})] + p_B[\gamma[1 - F(\sigma_{WB})] + (1 - \gamma)[1 - F(\sigma_{WA})]].$$  
(32)
For $\gamma > \gamma^M(l)$, it holds that $p_i - z_i = s$ for all $i = 1, 2$ by (12). Thus, the monopolist maximizes (32) over $(z_A, z_B, p_A, p_A + \Delta p)$ s.t. $p_i = s + z_i$ for all $i$. By substituting all constraints into (32), (32) becomes a function of $z_A$ and $\Delta p$ only. By continuity, for $\Delta p \geq 0$ sufficiently low, the first-order condition with respect to $z_A$ characterizes the optimal $z_A$.

\[
[(1 - F(\sigma_{WA})) + (1 - F(\sigma_{WB}))] - z_A \frac{f(\sigma_{WA}) + f(\sigma_{WB})}{(1 - \gamma)(1 + l\gamma)} - \Delta p \frac{(1 - \gamma)f(\sigma_{WA}) + \gamma f(\sigma_{WB})}{(1 - \gamma)(1 + l\gamma)} = 0.
\]

At $\Delta p = 0$, this equation elapses to (30). In addition, at $\Delta p = 0$, the first derivative of the monopolist’s profit with respect to $\Delta p$ equals

\[
-z_A f(\sigma_W) \left[ 1 - \frac{(1 - \gamma)(1 - l\gamma)}{(1 - \gamma)(1 + l\gamma)} \right].
\]

Because $z_A > 0$, this derivative is negative for $l > 0$ and zero only for $l = 0$, i.e., it is optimal to set $\Delta p = 0$.

It remains to consider the case where $\gamma \leq \gamma^M(l)$. In this case it holds that $p_i - z_i = s - [l\gamma(1 - \gamma) - \gamma + 1/2]\sigma_{wi}$ for all $i = 1, 2$ by (12). The monopolist maximizes (32) over $(z_A, z_B, p_A, p_A + \Delta p)$ s.t. $p_i = z_i + s - [l\gamma(1 - \gamma) - \gamma + 1/2]\sigma_{wi}$ for all $i$. Using the fact that $\sigma_{wi}$ only depends on $z_i$ and $\Delta p$, it follows from the constraints that $z_B = z_A + 2\Delta p(1 - \gamma)\gamma[(2\gamma - 1)l + 2]$. By substituting into (32), (32) becomes again a function of $z_A$ and $\Delta p$ only. For $\Delta p \geq 0$ sufficiently low, the first-order condition with respect to $z_A$ characterizes the optimal $z_A$. At $\Delta p = 0$, the first derivative of the monopolist’s profit with respect to $\Delta p$ can be expressed as follows (using the first-order condition with respect to $z_A$),

\[
-(1 - \gamma)l\gamma \left[ 1 - F\left( \frac{z_A}{(1 - \gamma)(1 + l\gamma)} \right) \right].
\]

This derivative is negative for $l > 0$ and zero only for $l = 0$, i.e., it is optimal to set $\Delta p = 0$.

\[\square\]

**Proof of Corollary 1.** The result is an immediate consequence of Proposition 1 and the fact that $\lim_{l \to 0} \gamma^M(l) = \frac{1}{2}$. \[\square\]

**Proof of Proposition 2.** Let $\sigma_\gamma \equiv \lim_{\gamma \to \frac{1}{2}} \sigma^*_W$ and $\sigma_1 \equiv \lim_{\gamma \to 1} \sigma^*_W$ be the equilibrium fraction of advance sales in the limits of an uninformed and an informed market, respectively.
For $\gamma \rightarrow \frac{1}{2}$ the first order condition (20) determining the equilibrium fraction of advance sales is given by

$$G_{\frac{1}{2}}(\sigma_W, l) = \frac{1 - F(\sigma_W)}{f(\sigma_W)} - \sigma_W + \frac{1}{2}(1 + \frac{l}{2})\sigma_W = 0. \quad (35)$$

As $f$ has an increasing hazard rate and $l < 2$, $G_{\frac{1}{2}}$ is strictly decreasing in $\sigma_W$, positive for $\sigma_W = 0$, and negative for $\sigma_W = 1$. Hence $\sigma_{\frac{1}{2}}$ is well defined as the unique solution to (35) in $(0, 1)$. As $G_{\frac{1}{2}}$ is strictly increasing in $l$, it follows that $\sigma_{\frac{1}{2}}$ is strictly increasing in $l$. Similarly, for $\gamma \rightarrow 1$, (20) becomes

$$G_1(\sigma_W, l) = \frac{1 - F(\sigma_W)}{f(\sigma_W)} - \sigma_W + \frac{1}{(1 + l)f(0)} = 0. \quad (36)$$

As $f$ has an increasing hazard rate, $G_1$ is strictly decreasing in $\sigma_W$. It is positive for $\sigma_W = 0$. It is negative for $\sigma_W = 1$ if and only if $\frac{1}{(1+l)f(0)} < 1 \Leftrightarrow \frac{1}{f(0)} - 1 < l$. If this condition holds, then $\sigma_1$ is given by the unique solution to (36) in $(0, 1)$. Otherwise $\sigma_1 = 1$. As $G_1$ is strictly decreasing in $l$, it follows that $\sigma_1$ is strictly decreasing in $l$ as long as $\sigma_1 < 1$. \hfill \Box

**Proof of Corollary 2.** Consider first the case without loss aversion, i.e. let $l = 0$. The functions $G_{\frac{1}{2}}(\sigma_W, 0)$ and $G_1(\sigma_W, 0)$ in (35) and (36), defining (implicitly) the equilibrium fraction of advance sales in the limits $\gamma \rightarrow \frac{1}{2}$ and $\gamma \rightarrow 1$, are both decreasing in $\sigma_W$ but do so at different rates. They intersect once at $\sigma_W = \tilde{\sigma} = \frac{2}{f(0)}$ and $G_1$ crosses $G_{\frac{1}{2}}$ from above, i.e. $G_{\frac{1}{2}}(\sigma_W, 0) < G_1(\sigma_W, 0)$ if and only if $\sigma_W < \tilde{\sigma}$. If $\frac{2}{f(0)} > 1$ then $\sigma_{\frac{1}{2}} < \sigma_1$. Consider the remaining case where $\frac{2}{f(0)} \leq 1$. In this case it holds that $\sigma_{\frac{1}{2}} < \sigma_1$ if and only if $G_{\frac{1}{2}}$ and $G_1$ intersect below the zero axis, that is, if and only if

$$\frac{1 - F(\tilde{\sigma})}{f(\tilde{\sigma})} - \frac{1}{2}\tilde{\sigma} < 0. \quad (37)$$

Note that

$$\frac{1 - F(\tilde{\sigma})}{f(\tilde{\sigma})} - \frac{1}{2}\tilde{\sigma} = \frac{1 - F(\frac{2}{f(0)})}{f(\frac{2}{f(0)})} - \frac{1}{f(0)} \leq \frac{1 - F(0)}{f(0)} - \frac{1}{f(0)} = 0 \quad (38)$$

where the inequality follows because $f$ has an increasing hazard rate. Hence we have shown that $\sigma_{\frac{1}{2}} \leq \sigma_1$ and that this inequality becomes strict when $f(0) < 2$ or when the hazard rate of $f$ is strictly increasing.
In the remainder, we determine a lower bound  \( \underline{l} \) on the consumers’ degree of loss aversion such that \( \sigma_1 < \sigma_{\frac{l}{2}} \) for all \( l > \underline{l} \). For this purpose, note that \( \sigma_1 < \sigma_{\frac{l}{2}} \) if and only if \( G_{\frac{l}{2}} \) and \( G_1 \) intersect at \( \tilde{\sigma} < 1 \) and this intersection lies above the zero-axis. More precisely, \( \sigma_1 < \sigma_{\frac{l}{2}} \) if and only if

\[
\tilde{\sigma}(l) = \frac{2}{(1 + l)(1 + \frac{l}{2})f(0)} < 1 \iff l > -\frac{3}{2} + \sqrt{\frac{1}{4} + \frac{4}{f(0)}} \equiv \tilde{l} \tag{39}
\]

and

\[
K(l) \equiv G_{\frac{l}{2}}(\tilde{\sigma}(l)) = \frac{1 - F(\tilde{\sigma}(l))}{f(\tilde{\sigma}(l))} - \left( \frac{1}{2} - \frac{l}{4} \right) \tilde{\sigma}(l) > 0. \tag{40}
\]

Note that \( \tilde{l} < 2 \) if and only if \( f(0) > \frac{1}{2} \). If \( K(\tilde{l}) > 0 \) then, because \( K(l) \) is strictly increasing, \( \sigma_1 < \sigma_{\frac{l}{2}} \) for all \( l \in (\tilde{l}, 2) \). If \( K(\tilde{l}) < 0 \) then the fact that \( K(2) > 0 \) implies that there exists a \( \tilde{\sigma} \in (\tilde{l}, 2) \) such that \( K(l) > 0 \) if and only if \( l > \tilde{\sigma} \). Defining \( \underline{l} \equiv \max\{\tilde{l}, \tilde{\sigma}\} < 2 \) we have therefore shown that \( \sigma_1 < \sigma_{\frac{l}{2}} \) if and only if \( l > \underline{l} \).

For the example of a uniform \( f \) we have \( \tilde{l} = \frac{\sqrt{17} - 3}{2} \) and with \( \tilde{\sigma} = \frac{2}{(1 + l)(1 + \frac{l}{2})} \) we get \( K(l) = 1 - \frac{3 - \frac{l}{2}}{(1 + l)(1 + \frac{l}{2})} \). From \( K(l) = 0 \) we find \( \tilde{l} = \sqrt{8} - 2 \). Hence, for \( f \) uniform \( \sigma_1 < \sigma_{\frac{l}{2}} \) if and only if \( l > \underline{l} = \max\{\frac{\sqrt{17} - 3}{2}, \sqrt{8} - 2\} = \sqrt{8} - 2 \approx 0.83 \).

**Proof of Proposition 3.** The monopolist’s profit maximizing prices in (23) and (24) have been derived in the main text and in the proof of Proposition 1. The comparative statics follow from the fact that

\[
\frac{\partial}{\partial l} \left[ \frac{ly(1 - \gamma) - \gamma + \frac{1}{2}}{4(1 - \gamma)(1 + ly)} \right] = \frac{\gamma}{8(1 - \gamma)(1 + ly)^2} > 0. \tag{41}
\]

**Proof of Corollary 3.** For \( l = 0 \) we have \( \frac{\partial}{\partial \gamma} [p^M - z^M] = \frac{\partial}{\partial \gamma} [s] = 0 \) and \( \frac{\partial}{\partial \gamma} [p^M] = \frac{\partial}{\partial \gamma} [s + \frac{1}{2}(1 - \gamma)] = -\frac{1}{2} \). If \( l > 0 \) then \( \gamma^M(l) \in (\frac{1}{2}, 1) \) and for all \( \gamma \in (\frac{1}{2}, \gamma^M(l)) \) it holds that

\[
\frac{\partial}{\partial \gamma} [p^M - z^M] = \frac{\partial}{\partial \gamma} [p^M] = -\frac{\partial}{\partial \gamma} \left[ \frac{ly(1 - \gamma) - \gamma + \frac{1}{2}}{4(1 - \gamma)(1 + ly)} \right] = \frac{4l(2\gamma - 1) + 1}{8(1 - \gamma)^2(1 + ly)^2} > 0. \tag{42}
\]
Proof of Lemma 1. The proof is presented in the main text. □

Proof of Lemma 2. From the first order condition (26) we get

\[ p^*(z) = \frac{(2\gamma - 1)(1 + l_\gamma) + z[1 + l_\gamma + (2\gamma - 1)(1 + l_m\gamma)]}{1 + l_\gamma + 2(1 - \gamma)(2\gamma - 1)}. \] (43)

Note that \( p^*(z) \) is increasing in \( l_m \). Moreover

\[ \frac{\partial p^*(z)}{\partial l_t} = \frac{\gamma(2\gamma - 1)[2(1 - \gamma)(2\gamma - 1) - z[2\gamma - 1 + l_m\gamma]]}{[1 + l_\gamma + 2(1 - \gamma)(2\gamma - 1)]^2} \] (44)

implies that \( p^*(z) \) is increasing in \( l_t \) when \( z < \frac{2(1 - \gamma)(2\gamma - 1)}{2\gamma - 1 + l_m\gamma} \) and decreasing when the opposite holds. □

Proof of Proposition 4. We first prove the comparative statics for \( p^* \) and \( z^* \). For this purpose, note that \( z^*(p) \) and \( p^*(z) \) are increasing functions. This follows from the fact that (25) and (26) are increasing in the price-level but decreasing in the discount. We consider the following cases: (1) \( l = l_t = l_m > 0 \); (2) \( l = l_m > 0 = l_t \); and (3) \( l = l_t > 0 = l_m \).

(1) For \( l = l_t = l_m > 0 \), (26) is decreasing in \( l \) which implies that \( p^*(z) \) is increasing in \( l \). Moreover, from Lemma 1 we know that \( z^*(p) \) is increasing in \( l \). As \( z^*(p) \) and \( p^*(z) \) are increasing functions, it follows that the equilibrium values \( z^* \) and \( p^* \) are both increasing in \( l \).

(2) When \( l = l_m > 0 = l_t \), Lemma 1 implies that \( z^*(p) \) is independent of \( l \). However, according to Lemma 2, \( p^*(z) \) is increasing in \( l \). As \( z^*(p) \) and \( p^*(z) \) are increasing functions, the equilibrium values \( p^* \) and \( z^* \) must be both increasing in \( l \).

(3) For \( l = l_t > 0 = l_m \), (28) simplifies to

\[ p^* = \frac{(\gamma l + 1)(1 - \gamma)(2\gamma l + 3\gamma - 1)}{\gamma[7 + 2l - 2(1 - \gamma)(2\gamma l - 4\gamma) - 1]}. \] (45)

Note that for all \( \gamma \in (\frac{1}{2}, 1) \) it holds that

\[ \frac{\partial p^*}{\partial l} = \frac{2\gamma(1 - \gamma)[(1 - \gamma)(1 - \gamma)]\gamma^2l^2 + [4\gamma(1 - \gamma) + 3\gamma - 1]\gamma l + (2\gamma - 1)^2 + 3\gamma^2}{\gamma[7 + 2l - 2(1 - \gamma)(2\gamma l - 4\gamma) - 1]^2} > 0. \] (46)
Moreover, from Lemma 1 we know that \( z^*(p) \) is increasing in \( l \). Because \( z^*(p) \) is an increasing function and \( p^* \) is increasing in \( l \), it must therefore hold that \( z^* = z^*(p^*) \) is increasing in \( l \).

It remains to prove the comparative statics for \( p^* - z^* \), because, as both \( p^* \) and \( z^* \) are increasing, the overall effect of loss aversion on the first period price is unclear. From (27) and (28) we get

\[
p^* - z^* = \frac{(1 + l\gamma)\gamma(1 + 2\gamma) - 1}{\gamma[7 + 2l - (1 - \gamma)\gamma(l + 2)l - 2\gamma(l - (1 - \gamma)l_m + 2)] - 1}.
\]

For \( l = l_m > 0 = l_t \) it is immediate that \( p^* - z^* \) is increasing in \( l \). For \( l = l_t > 0 = l_m \), \( p^* - z^* \) simplifies to

\[
p^* - z^* = \frac{(1 + l\gamma)\gamma(1 + 2\gamma) - 1}{\gamma[7 + 2l - (1 - \gamma)\gamma(l + 4)l - 4\gamma] - 1}
\]

and for all \( \gamma \in (\frac{1}{2}, 1) \) it holds that

\[
\frac{\partial (p^* - z^*)}{\partial l} = \frac{\gamma[2 - \gamma + (2\gamma - 1)(1 + 3(1 - \gamma)(1 + 2\gamma))]}{\gamma[7 + 2l - (1 - \gamma)\gamma(l + 4)l - 4\gamma] - 1}] > 0.
\]

Finally, for \( l = l_t = l_m > 0 \), we get

\[
p^* - z^* = \frac{(1 + l\gamma)\gamma(1 + 2\gamma) - 1}{\gamma[7 + 2l - (1 - \gamma)\gamma(l + 4)l - 4\gamma] - 1}
\]

and

\[
\frac{\partial (p^* - z^*)}{\partial l} = \frac{\gamma(1 - \gamma)(2\gamma - 1)(1 + \gamma)[(1 + l\gamma)^2 + 4(2\gamma - 1)]}{\gamma[7 + 2l - (1 - \gamma)\gamma(l + 4)l - 4\gamma] - 1}] > 0
\]

for all \( \gamma \in (\frac{1}{2}, 1) \).

\[\Box\]

**Proof of Corollary 4.** For \( l = 0 \) we have

\[
\frac{\partial p^* - z^*}{\partial \gamma} = \frac{6(1 - 2\gamma + 3\gamma^2)}{(1 - 7\gamma + 4\gamma^2)^2} > 0
\]

\[
\frac{\partial p^*}{\partial \gamma} = \frac{4(1 - 2\gamma + 3\gamma^2)}{(1 - 7\gamma + 4\gamma^2)^2} > 0
\]

for all \( \gamma \in (\frac{1}{2}, 1) \). To see that for \( l > 1 \), \( p^* \) is decreasing in \( \gamma \) for large \( \gamma \) note that

\[
\lim_{\gamma \to 1} \frac{\partial p^*}{\partial \gamma} = \frac{2 - l - l^2}{1 + l}
\]

is negative if and only if \( l > 1 \).

\[\Box\]
References


